DOI: 10.35598/mcfpga.2026.031

Method of Multilayer Dielectric Defect Parameteres Definition on Thermal Quadrupoles Basis

Olga Zaichenko
ORCID 0000-0003-4936-2785
Department of Design and Operation of Electronic Devices
Kharkiv National University of Radioelectronics
Kharkiv, Ukraine
Email olha.zaichenko@nure.ua

Nataliia Hapon
ORCID 0000-0001-9798-7136
Department of Design and Operation of Electronic Devices
Kharkiv National University of Radioelectronics
Kharkiv, Ukraine
Email nataliia.zaichenko@nure.ua

Abstract—The methods of multilayer dielectric defect parameteres definition on thermal quadrupoles basis are considered in the report. Analytical method for determining the defect depth and thicknesses or thermal resistance are proposed. Objective is to improve the accuracy and simplify the algorithm for calculating layer thicknesses. The results are obtained for the depth of the defect by determining the cotangent and for the thicknesses of the defect or thermal resistance by the basic trigonometric identity.

Keywords— thermal quadrupole, layer thickness, heat transfer, model

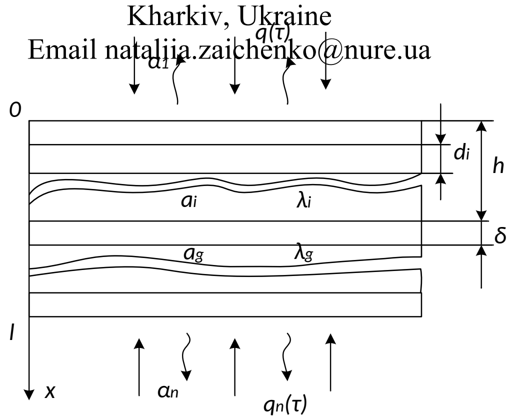
I. INTRODUCTION

Modern research in the field of thermal processes in electronic systems increasingly relies on mathematical models that allow describing the distribution of temperature and heat flows in complex multilayer structures [1-2]. One of the most effective tools for such analysis is the use of the concept of thermal quadrupoles, which allow representing heat transfer in a form similar to electrical circuits. Due to this, thermal processes can be analyzed using the methods of linear system theory, which greatly simplifies modeling and allows for parametric studies of the system. The matrix representation of thermal quadrupoles is based on the use of vector-matrix equations that relate temperatures and heat flows on different surfaces of the body through matrices of thermal characteristics. This approach provides universality of description - regardless of the geometry, material or number of layers of the system. In the case of a homogeneous medium limited by a closed surface, this allows describing the processes of stationary heat transfer through the ratio between the input and output flows, similar to the transfer of current and voltage in electrical circuits.

In this work we determine the depth and thickness of the defect.

II. MODEL OF MULTILAYER STRUCTURE AS THERMAL QUADRUPOLES

Structure and description of a multilayer structure. In order to obtain a model of a multilayer structure consider fig. 1.



III. FIG.1. MODEL OF A MULTILAYER STRUCTURE

A vertical axis on the left represents the spatial coordinate x, which points downwards. The top boundary of the multilayered structure is located at x = 0, and the bottom boundary is marked at x = l. drepresents the thickness of the top layers. δ represents the thickness of a defect internal layer or inclusion. h measures the depth from the top surface (x = 0) to the upper boundary of the defect layer.

The fig.1 depicts a composite or structure consisting of several horizontal layers. top and bottom outer layers: The system features regular, flat parallel layers near the outer boundaries. The region above the defect layer is characterized by thermal diffusivity a_i and thermal conductivity λ_i. The region below the defect layer is characterized by thermal diffusivity a_d and thermal conductivity λ_d. A layer of thickness δ is emphasized with diagonal hatching (shading), representing a defect material.

IV. METHOD OF MULTILAYER DIELECTRIC DEFECT PARAMETERES DEFINITION

The model was adopted from [3]; however, a feature of model [3] is that it requires matrix inversion, what we will try to avoid. The thermal resistance approach from [4] was utilized instead of modeling the defective layer directly. Also in [4] a defect-free sample is used as an additional connection between the quantities.

The proposed hybrid model is [5-6]

$$a = \begin{bmatrix} a_1 & a_4 \\ a_2 & a_3 \end{bmatrix} = \begin{bmatrix} \cos(kd) & -\sin(kd) \\ \sin(kd) & \cos(kd) \end{bmatrix} \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \times$$

$$\times \begin{bmatrix} \cos(kh) & -\sin(kh) \\ \sin(kh) & \cos(kh) \end{bmatrix}, \qquad (1)$$

where R is thermal resistance for defect.

As a result, we obtain for the element of the resulting matrix by equating the left and right sides of expression (1).

$$a_1 = \cos(kd)\cos(kh) - \sin(kd)\sin(kh) + R\cos(kd)\sin(kh), \ (2)$$

$$a_2 = \cos(kd)\sin(kh) + \cos(kh)\sin(kd) + R\sin(kd)\sin(kh), \ (3)$$

$$a_3 = \cos(kd)\cos(kh) - \sin(kd)\sin(kh) + R\cos(kh)\sin(kd), \ (4)$$

$$a_4 = R\cos(kd)\cos(kh) - \cos(kh)\sin(kd) - \cos(kd)\sin(kh). \ (5)$$

Let's assume that there is a defect-free sample, for this sample model is

$$a_0 = \begin{bmatrix} \cos(kd) & -\sin(kd) \\ \sin(kd) & \cos(kd) \end{bmatrix} \begin{bmatrix} \cos(kh) & -\sin(kh) \\ \sin(kh) & \cos(kh) \end{bmatrix} \qquad (6)$$

After performing the multiplication we get

$$a_0 = \begin{bmatrix} a_{01} & a_{04} \\ a_{02} & a_{03} \end{bmatrix} = \begin{bmatrix} \cos(kd+kh) & -\sin(kd+kh) \\ \sin(kd+kh) & \cos(kd+kh) \end{bmatrix}$$

(7)

As a result, we obtain for the element of the resulting matrix(7)

$$a_{01} = \cos(kd+kh) = \cos(kd)\cos(kh) - \sin(kd)\sin(kh), \ (8)$$

$$a_{02} = \sin(kd+kh) = \cos(kd)\sin(kh) + \cos(kh)\sin(kd), \ (9)$$

$$a_{03} = \cos(kd+kh) = \cos(kd)\cos(kh) - \sin(kd)\sin(kh), \ (10)$$

$$a_{04} = -\sin(kd+kh) = -\cos(kh)\sin(kd) - \cos(kd)\sin(kh). \ (11)$$

A. Defect Depth

From the definition of arctangent and equations (2)-(3) and (8)-(9)

$$kd = arcctg\frac{a_1 - a_{01}}{a_2 - a_{02}}. \qquad (12)$$

B. Thermal resistance

From difference of equation (3) and (9) expresses $\sin(kh)$, and squaring it, than difference of equation (5) and (11) express $\cos(kh)$ and squaring it too, and summarize them to get the basic trigonometric identity which is equal to one, let's express R under condition of known kd from (12)

$$R = \sqrt{\left(\frac{a_2 - a_{02}}{\sin(kd)}\right)^2 + \left(\frac{a_4 - a_{04}}{\cos(kd)}\right)^2}. \qquad (13)$$

V. Conclusion

The heat transfer processes in multilayer structures represented as equivalent thermal quadrupoles was considered in the report. Analytical method for determining the geometric parameters (thicknesses) of layers in thermal quadrupole was proposed. Objective is to improve the accuracy and simplify the algorithm for calculating layer thicknesses. The results are obtained for the depth of the defect by determining the cotangent and for the thicknesses of the defect or thermal resistance by the basic trigonometric identity.

References

[1] S.-M. David, J.-C. Batsale, J. Rodriguez-Aseguinolaza. “3D thermal quadrupoles for solving the defect detection problem in lock-in thermography,” Quantitative InfraRed Thermography Journal, 2026, 23.2, pp. 69-85.

[2] S.-M. David, J. Rodríguez-Aseguinolaza, J.-C. Batsale. “3D flaw geometry prediction from infrared thermography data using a frugal physics-aided artificial intelligence,” International Journal of Thermal Sciences, 2026, 225 , p.110787.

[3] V. A.. Storozhenko, S. I.. Melnik, ”Transfer function method in thermal defectometry,” Non-destructive testing, 1991, 12, pp. 78–83.

[4] D. Maillet, A. S.Houlbert, S. Didierjean, A. S.Lamine, A. Degiovanni, “Non-destructive thermal evaluation of delaminations in a laminate: Part II—The experimental Laplace transforms method,” Composites science and technology, 1993,. 47(2), pp.155-172.

[5] O. Zaichenko, N. Hapon, I. Khoroshailo, “Improved Thermal Quadrupoles Model for Multilayer Dielectric with Defect,” In 2025 IEEE 6th KhPI Week on Advanced Technology (KhPIWeek), 2025, October, pp. 1-4.

[6] O. Zaichenko, N. Hapon, “Implementation of Algorithm of Thermal Quadrupoles for Filament Defect Detection,” VII International Scientific and Practical Conference Theoretical and Applied Aspects of Device Development on Microcontrollers and FPGA(MC&FPGA-2025). – 2025. – C. 29–30.